Analog MIMO detection on the basis of Belief Propagation

Josep Soler-Garrido, Robert J. Piechocki
and Koushik Maharatna
Centre for Communications Research
University of Bristol
Bristol, UK
Email: Josep.Soler@bristol.ac.uk

Darren McNamara
Telecommunications Research Laboratory
Toshiba Research Europe Ltd.
Bristol, UK

***Abstract*— In this article we present the design of an analog detector for Multiple-Input Multiple-Output (MIMO) wireless systems, based on the well known Belief Propagation (BP) algorithm. BP has been shown to obtain excellent results when solving inference problems in sparsely connected factor graphs. Unfortunately, MIMO detection is an example of inference in a densely connected graph, so other techniques need to be applied. We show that BP in conjunction with simulated annealing can offer near optimal performance when there is enough diversity in the receiver. Interestingly, this algorithm can be easily mapped into analog circuitry, thus leading to potentially low-power area-efficient MIMO detectors. A small analog detector based on this work has been designed and laid out in a 0.25µm BiCMOS process.**

I. INTRODUCTION

Analog signal processing is an emerging topic with applications in an increasing number of fields. In the field of communications, the lower power consumption that can be achieved with analog circuits makes them an attractive alternative to their digital counterparts for signal processing tasks in portable devices.

Analog decoding of channel codes is a good example of this new philosophy of receiver design. The natural way in which decoding algorithms are mapped into analog circuits [1,2] resulted in the fabrication of analog channel decoders reporting an improvement of several orders of magnitude in power consumption, silicon area or processing speed compared to digital designs.

Many of these circuits implement the sum-product or Belief Propagation (BP) algorithm [3]. This algorithm can be used to decode the most powerful channel codes such as LDPC [4] and turbo codes [5]. In these schemes the decoding process takes place on a factor graph where messages are propagated between different nodes of a network. However, these algorithms cannot be directly applied to other communication problems such as MIMO or CDMA multiuser detection due to the fully connected nature of their factor graphs. In these scenarios, BP-related algorithms typically fail to converge, or if they do, the quality of the estimations provided is far from optimal.

It is the aim of this paper to present a first practical method of MIMO detection based on BP, which can therefore be easily implemented by using well known analog decoding structures. This work brings us closer to the design of turbo-MIMO receivers in which MIMO detection and channel decoding take place in continuous time in an analog network.

The rest of the paper is organized as follows: section II introduces spatial multiplexing systems and detection algorithm, section III deals with its analog implementation, and section IV presents the first implemented analog MIMO detector and provides some initial results. Conclusions are drawn in section V.

II. SYSTEM DESCRIPTION AND DETECTION ALGORITHM

A. *System Description.*

Let us consider a coded MIMO system with N_T transmit antennas and N_R receive antennas, as the one depicted in Fig. 1. At the transmitter side, K information bits $b_{1:K}$ are first encoded to N coded bits $c_{1:N}$, randomly interleaved, modulated and mapped onto all N_T transmit antennas. The transmission of the resulting vector $x_{1:NT}$ takes place over a narrowband channel represented by a matrix $\mathbf{H}$ of size $N_R \times N_T$, where each entry $h_{i,j}$ defines a channel connecting transmit antenna j and receive antenna i. This scheme is usually referred to as V-BLAST or Spatial Multiplexing [9]. The system is typically modelled as:

$$\mathbf{y} = a\mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is the received vector of length N_R, a is a normalization constant that makes the total energy per symbol equal to unity, and the vector $\mathbf{n}$ accounts for the Gaussian noise: $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2\mathbf{I})$.

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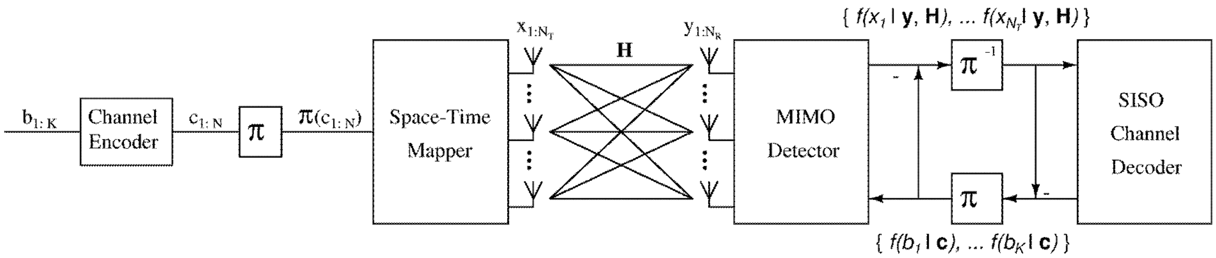


Figure 1. Diagram of a coded MIMO system.

The receiver will try to retrieve the information bits from the received values. Assuming knowledge of the channel matrix $\mathbf{H}$, its specific task will be to obtain the full set of marginal posterior probabilities (MPPs) given by:

$$\{f(b_1|\mathbf{y},\mathbf{H}), f(b_2|\mathbf{y},\mathbf{H}), \ldots, f(b_K|\mathbf{y},\mathbf{H})\}.$$

The high complexity of this task is usually overcome by using so-called turbo detection, a sub-optimal iterative method in which a MIMO detector computes

$$\{f(x_1|\mathbf{y},\mathbf{H}), f(x_2|\mathbf{y},\mathbf{H}), \ldots, f(x_{N_T}|\mathbf{y},\mathbf{H})\},$$

and a channel decoder obtains

$$\{f(b_1|c_{1:N}), f(b_2|c_{1:N}), \ldots, f(b_K|c_{1:N})\},$$

and both independent decoders exchange their extrinsic information in an iterative process that usually converges to an improved estimation.

Since analog implementation of the channel decoder block has been extensively studied, in the rest of the paper we will focus on the implementation of the MIMO detector part of the receiver.

B. Optimal detection.

The optimal Maximum a Posteriori (MAP) detector enumerates all the elements of the joint posterior distribution

$$f(\mathbf{x}|\mathbf{y},\mathbf{H}) \propto f(\mathbf{y}|\mathbf{x},\mathbf{H}) \cdot f(\mathbf{x}) \quad (2)$$

and then marginalizes out each variable:

$$f(x_i|\mathbf{y},\mathbf{H}) = \sum_{x_{-i}} f(\mathbf{x}|\mathbf{y},\mathbf{H}) \quad (3)$$

where x_{-i} stands for "all entries of $\mathbf{x}$ except x_i". Given the Gaussian likelihood and considering BPSK modulation (i.e. $x_i \in [-1,1]$) the joint posterior distribution can be expressed as

$$f(\mathbf{x} \mid \mathbf{y}, \mathbf{H}) = \prod_i \phi_i(x_i) \cdot \prod_{(i>j)} \varphi_{ij}(x_i, x_j) \quad (4)$$

where:

$$\phi_i(x_i) = \exp\left[x_i z_i + \log\ p(x_i)\right] \quad (5)$$

$$\varphi_{i,j}(x_i, x_j) = \exp(-x_i \cdot x_j R_{i,j}). \quad (6)$$

The values of $\mathbf{R}$ and $\mathbf{z}$ are given by the channel's cross-correlation matrix and the output of the matched filter $\mathbf{H}^H$:

$$\mathbf{R} = \frac{a^2}{\sigma_n^2} \mathbf{H}^H \mathbf{H} \quad (7)$$

$$\mathbf{z} = \frac{a}{\sigma_n^2} \mathbf{H}^H \mathbf{y} \,. \quad (8)$$

MAP detection has a complexity that grows exponentially with the number of transmit antennas, so it becomes intractable even for configurations of moderate size. Although analog implementation of this optimal detector has been proposed [11], its applicability is reduced to simple MIMO configurations, so in this paper we opt for a sub-optimal algorithm described in the following subsection.

C. Proposed detection algorihm.

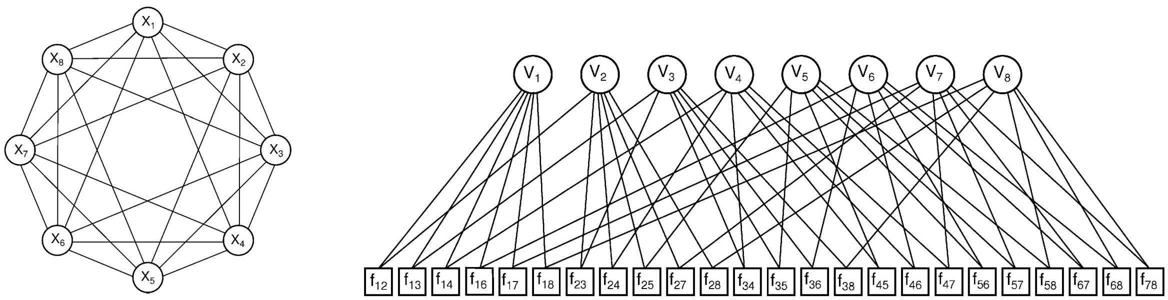
In order to reduce the complexity of the decoder, approximate algorithms can be applied. The values of ϕ and φ given by equations (5) and (6) define respectively the variable and edge potentials of an undirected graphical model to which we can apply message-passing algorithms, such as belief propagation, in order to lower the computational cost of detection.

As aforementioned, the main problem we face is that these algorithms tend to perform poorly in highly connected graphs, such as the one depicted in Fig. 2(a), which represents a 4-antenna MIMO system with QPSK modulation. In this particular graph, every variable x_i is connected to the rest of variables except for $x_{\mathrm{mod}(i+N_T-1,2N_T)+1}$ due to the orthogonality of the in-phase and quadrature components of symbols transmitted from the same antenna.

Belief propagation tries to estimate the marginal probabilities for all variables by means of passing messages between local nodes. The local belief at every node is calculated as the product of the local evidence and all incoming messages [3]:

$$b(x_i) \propto \phi_i(x_i) \prod_j m_{ji}(x_i), \quad (9)$$

where the message from node j to node i, $m_{ji}(x_i)$, represents the state in which x_i should be according to node j.



(a) (b)

Figure 2. (a) Undirected graphical model for a QPSK MIMO system with N_T=4, (b) Factor graph representation.

The messages in belief propagation are obtained as:

$$m_{ji}(x_i) = \sum_{x_j} \phi_j(x_j)\varphi_{ji}(x_j, x_i) \prod_{k \neq i} m_{kj}(x_j). \quad (10)$$

This message update rule results in optimal inference in graphs without loops, but also can provide excellent results for sparse loopy graphs. The mechanism is best represented in a factor graph [6] such as the one of Fig. 2(b). The factor graph contains two types of nodes: variable (circles) and function (squares) nodes, and messages are passed along the edges. In our particular binary case and according to (10), the expressions for the messages are:

$$m_{V_i \to f_{ij}} = c \begin{bmatrix} \phi_i(1) \cdot \prod_{k \neq j} m_{f_{i,k} \to V_i}(1) \\ \phi_i(-1) \cdot \prod_{k \neq j} m_{f_{i,k} \to V_i}(-1) \end{bmatrix} \quad (11)$$

$$m_{f_{i,j} \to V_i} = c \begin{bmatrix} m_{V_j \to f_{i,j}}(1) \cdot \varphi_{i,j}(1,-1) + m_{V_j \to f_{i,j}}(-1) \cdot \varphi_{i,j}(1,1) \\ m_{V_j \to f_{i,j}}(1) \cdot \varphi_{i,j}(-1,-1) + m_{V_j \to f_{i,j}}(-1) \cdot \varphi_{i,j}(-1,1) \end{bmatrix}, (12)$$

where the constant c (different in both expressions) makes the sum of the elements of the messages equal to unity, since they represent binary probability distributions.

Unfortunately, and as previously explained, belief propagation does not perform well in the MIMO graphs due to the high density of connections. To tackle this problem we can resort to simulated annealing [7], a generic algorithm for global optimization problems.

The annealing or 'temperature' parameter will be the noise variance σ_n^2, which will be set to a high value any time a new symbol is to be detected, and will be allowed to gradually decrease. When the 'cooling down' process is finished, the variables will hopefully have settled to a minimum energy state that will give a good approximation to the true marginal probabilities. This method has been also used in conjunction with variational techniques such as Mean Field to solve multi-user detection problems in CDMA [8].

Annealing can be applied directly to the inputs of BP algorithm, so we can rewrite equations (7) and (8) as:

$$\hat{\mathbf{z}} = \frac{a}{T(t)} \mathbf{H}^H \mathbf{y} \quad (13)$$

$$\hat{\mathbf{R}} = \frac{a^2}{T(t)} \mathbf{H}^H \mathbf{H}, \quad (14)$$

where $T(t)$ is a time-varying temperature parameter. Alternatively, it can be applied to the messages in the factor graph, although the benefit is minimal and the complexity is increased.

One of the main drawbacks of simulated annealing is its slow convergence. We can expect that more iterations will be needed than with simple Belief Propagation. In the analog circuit this will translate into a longer transient response before the output currents have settled to the correct probabilities. Fig. 3 shows the bit-error-rate (BER) curves for our 4-antenna QPSK system using MAP, BP, and Annealing BP algorithms. The results are presented for three different configurations using 4, 6 and 8 receive antennas respectively. It can be seen how the approximate algorithms perform poorly when there is no spatial diversity, but the system becomes more orthogonal and the performance is improved when the number of receive antennas increases.

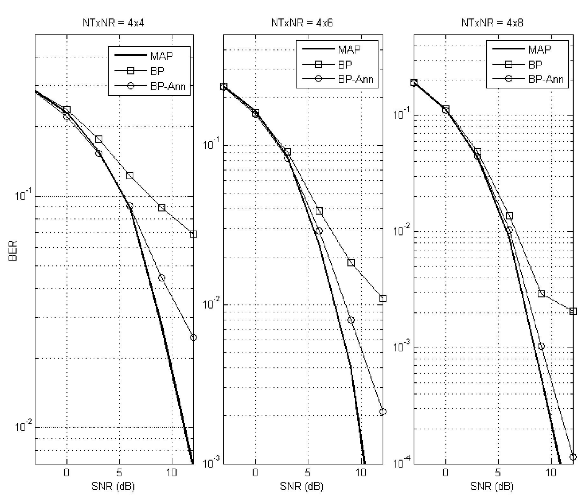


Figure 3. Simulation results for the uncoded system.

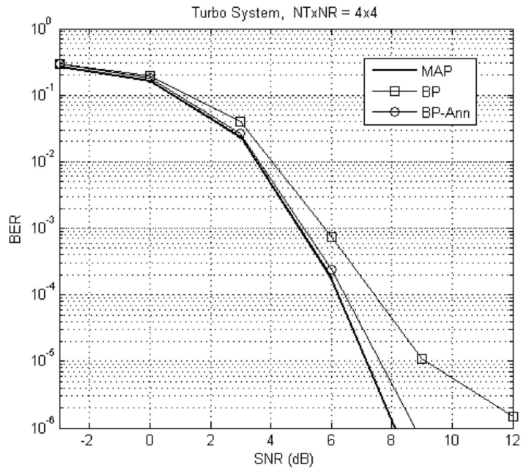


Figure 4. Simulation results for a coded turbo system.

However, near optimal performance can be achieved not only with spatial diversity, but also with temporal diversity such as the one provided by a fast fading channel, or frequency diversity as in a MIMO-OFDM system. Fig. 4 shows BER simulations for the whole system of Fig.1 with $N_T = N_R = 4$, in a fast-fading environment, after one single turbo iteration and employing a rate ½ convolutional encoder and random interleaver. The high degree of temporal diversity makes our algorithm perform closely to MAP detection.

III. ANALOG IMPLEMENTATION

Analog implementation of BP and related algorithms was first proposed in [1], leading to the design of analog channel decoders based on the perfect match between probability computations and translinear circuits. The most common topology is based on a generalized Gilbert multiplier circuit depicted in Fig. 5(a), able to obtain the product of two probability mass functions, by exploiting the exponential voltage-current characteristic of bipolar and subthreshold MOS transistors. After analysis of the circuit, the following expression for the top currents is obtained:

$$I_{Zij} = \frac{I_{Xi} \cdot I_{Yj}}{\sum_{k=1}^{n} I_{Yk}}. \tag{15}$$

If each set of input currents represents a probability mass function (i.e. the sums of all elements is equal to a reference current equivalent to probability one), the output currents can be interpreted as the different elements of the product distribution. From this generic circuit, analog cells for BP operations have been successfully developed [1]. By looking at equations (11) and (12), it is clear that only two different probability operations are needed in order to implement BP algorithm in our case:

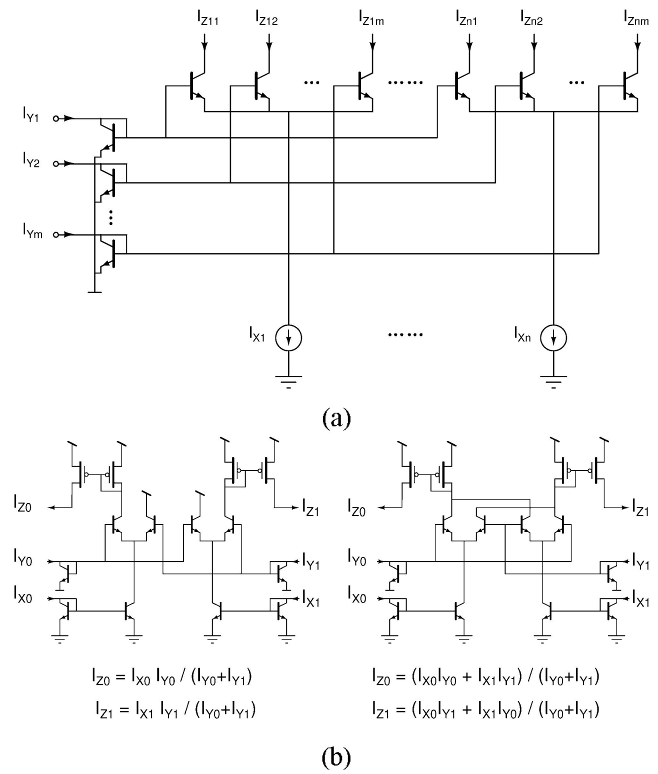


Figure 5. (a) Generic probability multiplier, (b) Analog probability gates: soft-equal and soft-XOR.

$$\begin{bmatrix} p_Z(0) \\ p_Z(1) \end{bmatrix} = \begin{bmatrix} p_X(0) \cdot p_Y(0) \\ p_X(1) \cdot p_Y(1) \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} p_Z(0) \\ p_Z(1) \end{bmatrix} = \begin{bmatrix} p_X(0) \cdot p_Y(0) + p_X(1) \cdot p_Y(1) \\ p_X(0) \cdot p_Y(1) + p_X(1) \cdot p_Y(0) \end{bmatrix}. \tag{17}$$

These two operations are respectively known as soft-equal and soft-XOR and their circuit implementation, shown in Fig. 5(b), has been widely used by the analog decoding community. The fact that our MIMO detector is constructed with the same circuits opens the door to future turbo receivers where channel decoding and MIMO detection are implemented on a single analog chip.

The remaining part of the design is to take care of the simulated annealing by just adding multiplier circuits between the inputs and the belief propagation core, as depicted in Fig. 6. We used a common single-ended annealing line for all the inputs, which is converted to differential and applied to Gilbert multiplier circuits. The voltage input will vary in time in order to simulate annealing in the belief propagation core. It is worth noting that different multiplier topologies may be needed depending on the particular design, especially if the inputs require a wide dynamic range.

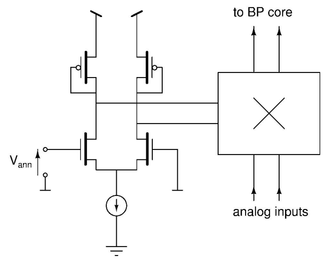


Figure 6. Simulated annealing set-up.

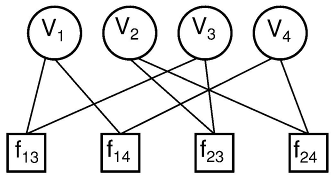


Figure 7. Implemented factor graph.

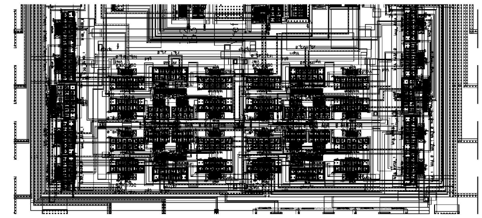


Figure 8. Layout of the analog MIMO detector.

IV. Example Detector

A simple proof-of-concept analog MIMO detector has been implemented in 0.25μm BiCMOS technology. The factor graph of this decoder is shown in Fig. 7, and corresponds to a 2-antenna QPSK spatial-multiplexing system, although it can also decode some particular Space-Time Block Code (STBC) arrangements (e.g. two parallel BPSK Alamouti [10] transmitters).

A bias current of 100μA per block was used. Although particular effort was made to obtain good device matching in detriment of decoding speed and silicon area, it is expected that the circuit can work at up to 30Mbps with an estimated power consumption of a few tens of milliwatts. The resulting layout is depicted in Fig. 8.

An example of the function of the circuit is presented in Fig. 9 with a short transient simulation, in which 4 different symbols are mixed by random channel values and then decoded by the analog circuit.

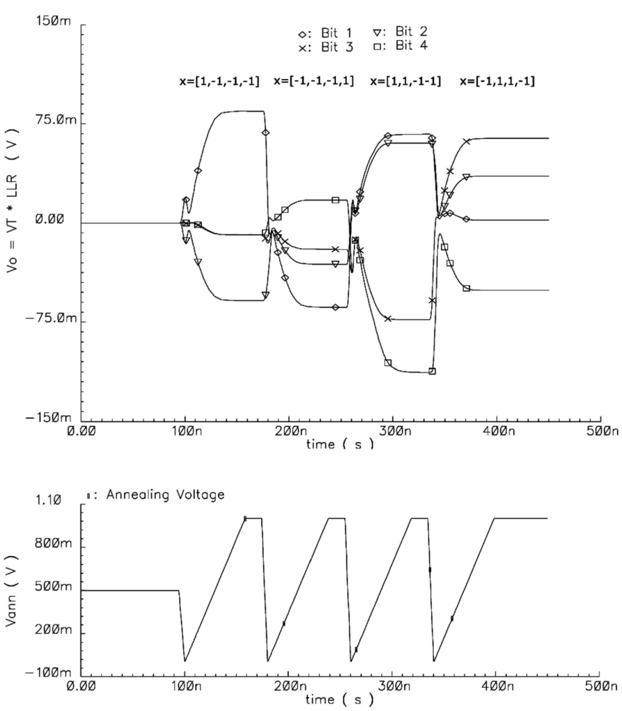


Figure 9. Transient simulation example.

The top graph represents the output marginal probabilities of all four bits in log-likelihood ratio (LLR) representation. The annealing voltage is represented in the bottom graph. Once the chip is fabricated, effects of transistor mismatch and different annealing schedules will be analyzed on real silicon.

V. Conclusions

A reduced complexity method for detection in MIMO systems has been presented. Analog implementation has been proposed based on the analog circuits previously applied to channel decoding. This work is aimed to represent a first step towards a multi-antenna wireless receiver where the signal processing tasks are carried out in continuous time by low-power analog circuitry.

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